Geometry, Topology and Computation in Groups

Schedule

Monday, March 8:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Pierre de la Harpe**: *A propos de Max Dehn, de topologie et de théorie des groupes.*
9:50 - 10:10	coffee
10:10 - 11:00	**Martin Bridson**: *Groups with quadratic Dehn functions (I).*
11:10 - 12:00	**Denis Osin**: *Filling in groups acting on hyperbolic spaces (I).*
12:15	lunch
13:00 - 17:00	ski / free time
17:00 - 18:00	coffee at Hôtel les Sources
18:00 - 18:50	**Rostislav Grigorchuk**: *Torsion images of Coxeter groups and the Wiegold problem.*
19:00	dinner

Tuesday, March 9:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Martin Bridson**: *Groups with quadratic Dehn functions (II).*
9:50 - 10:10	coffee
10:10 - 11:00	**Denis Osin**: *Filling in groups acting on hyperbolic spaces (II).*
11:10 - 12:00	**Laurent Bartholdi**: *Automatically presented groups and algebras.*
12:15	lunch
13:00 - 17:00	ski / free time
17:00 - 18:00	coffee at Hôtel les Sources
18:00 - 18:50	**Ilya Kapovich**: *Geometry of $Out(F_n)$ and geodesic currents on free group.*
19:00	dinner

Wednesday, March 10:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Sarah Rees**: *The Word Problem in the Chomsky Hierarchy (I).*
9:50 - 10:10	coffee
10:10 - 11:00	**Alexei Miasnikov**: *tba (I).*
11:10 - 12:00	**Olga Kharlampovich**: *Interesting examples of solvable groups.*
12:15	lunch
13:00 - 17:00	ski / free time
17:00 - 18:00	coffee at Hôtel les Sources
18:00 - 18:50	**Volker Diekert**: *On Computing Geodesics in Baumslag-Solitar Groups.*
19:00	dinner

Thursday, March 11:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Patrick Dehornoy**: *Le probleme de mots pour les groupes de tresses.*
9:50 - 10:00	coffee
10:10 - 11:00	**Martin Bridson**: *Groups with quadratic Dehn functions (III).*
11:10 - 12:00	**Denis Osin**: *Filling in groups acting on hyperbolic spaces (III).*
12:15	lunch
13:00 - 17:00	ski / free time
17:00 - 18:00	coffee at Hôtel les Sources
18:00 - 18:50	**Francois Dahmani**: *Hyperbolic geometry to solve instances of Dehn's third problem, the isomorphism problem.*
19:00	dinner

Friday, March 12:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Bettina Eick**: *The classification of p-groups by coclass.*
9:50 - 10:00	coffee
10:10 - 11:00	**Sarah Rees**: *The Word Problem in the Chomsky Hierarchy (II).*
11:10 - 12:00	**Alexei Miasnikov**: *tba (II).*
12:15	lunch

Abstracts of minicourses

Martin Bridson (Oxford): *Groups with quadratic Dehn functions.*

I shall begin by sketching a proof of the basic equivalence between the problem of filling loops in the universal cover of a closed manifold and the word problem for the fundamental group of the manifold. I shall explain the geometry behind the Brady-Bridson snowflake construction, proving that Dehn functions are dense in the super-quadratic range. I'll then concentrate on the class of finitely presented groups that satisfy a quadratic isoperimetric inequality, surveying the groups that are known to lie in this class and discussing what properties they have in common. I plan in particular to discuss what is known for lattices in semisimple Lie groups and what is known about the homological finiteness of groups with a quadratic Dehn function. The third lecture will include an explanation of the main ideas that go into proving that all free-by-cyclic groups satisfy a quadratic isoperimetric inequality.

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Denis Osin (Vanderbilt): *Filling in groups acting on hyperbolic spaces.*

We will start with a brief discussion of equivalent definitions and basic properties of relatively hyperbolic groups. We will then discuss a generalization of relative hyperbolicity based on the notion of a hyperbolically embedded subgroup. Examples of such subgroups naturally occur when a group admits a 'nice' action on a hyperbolic space. Finally we will show how hyperbolically embedded subgroups can be used to generalize the Gromov-Thurston theory of Dehn filling in hyperbolic 3-manifolds. Some applications to hyperbolic and relatively hyperbolic groups, mapping class groups, and outer automorphism groups of free groups will be presented.

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Sarah Rees (Newcastle): *The Word Problem in the Chomsky Hierarchy.*

I shall talk about the word problem for groups, and how hard it is to solve it. I'll view the word problem of $G = \langle X \rangle$ as the recognition of the set WP(G,X) of words over X that represent the identity, and look to relate properties of G to the complexity of WP(G,X) as a formal language, that is to the complexity of the Turing machine (model of computation) needed to recognize that set.

An elementary result identifies a group as finite precisely if its word problem is a regular language (recognised by a finite state automaton). A well known result of Muller and Schupp proves a group to be virtually free precisely if its word problem is context-free (recognised by a pushdown automaton), while Thomas and Herbst classify virtually cyclic groups using one counter automata. So at this bottom end of the Chomsky hierarchy we have complete classification results. I shall survey these briefly.

Many more groups are admitted if the word problem is allowed to be co-context-free, such as abelian groups, but no nilpotent groups that are not virtually abelian, various wreath product groups and the Houghton groups. I shall report on various results by myself, Holt, Röver, Lehnert and Schweitzer, and the techniques used to achieve them.

Many groups can be shown to have word problem that is soluble in linear space, and hence context-sensitive. In a second lecture I shall look at this class of groups, and at the subclasses with word problem that is growing context-sensitive or can be solved on a real-time Turing machine. In particular I'll examine Dehn's linear time algorithm for hyperbolic groups, which is growing context-sensitive, and can actually be programmed in real time. And I'll examine Cannon's generalisation of Dehn's algorithm, which allows the word problem for nilpotent groups to be solved in real time. I'll illustrate with examples of groups in all these classes, and results that separate the classes, referring to results of myself, Holt, Röver, Goodman, Shapiro, Kambites and Otto.

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Alexei Miasnikov (McGill U.): *tba.*

Abstracts of talks

Pierre de la Harpe (Geneva): *A propos de Max Dehn, de topologie et de théorie des groupes.*

Rostislav Grigorchuk (Texas A& M): *Torsion images of Coxeter groups and the Wiegold problem.*

Laurent Bartholdi (Göttingen): *Automatically presented groups and algebras.*

Ilya Kapovich (UIUC): *Geometry of $Out(F_n)$ and geodesic currents on free group.*

Olga Kharlampovich (McGill U.): *Interesting examples of solvable groups.*

I am going to discuss algorithmic problems for solvable groups, describe my construction of a f.p. 3-step solvable group with undecidable word problem, discuss interesting properties of this group and other interesting examples of solvable groups.

Volker Diekert (Stuttgart): *On Computing Geodesics in Baumslag-Solitar Groups.*

The Baumslag-Solitar group $BS(p,q)$ is a one-relator group defined by $BS(p,q) = \langle a, t \mid ta^pt^{-1} = a^q \rangle$. In my lecture I will give a dynamic programming method for computing geodesic words. As a consequence of our method we will see that if p divides q, then the set of horocyclic elements (elements in the cyclic subgroup $\langle a \rangle$) in length-lexicographical normal form is a deterministic (and unambiguous) linear context-free (one-counter) language; and it can be recognized in log-space. The growth series of the horocyclic subgroup is therefore a rational function (quotient of two polynomials) and can be calculated effectively. Rationality of the growth series is due to Freden et al. who showed it with a different method. The growth tells us how many group elements are in balls are in balls of a certain radius around the origin of the Cayley graph of $BS(p, q)$. If p divides q our main result is a square-time algorithm to compute the geodesic length of all group elements. This is a positive partial answer to a question raised by Elder et al. in 2009. In the case where p does not divide q it might be that the problem is actually Co-NP-complete. The investigation of the general case remains a challenging research project. My lecture is based on a joint work with Jurg Laun.

Patrick Dehornoy (Caen): *Le probleme de mots pour les groupes de tresses.*

Francois Dahmani (U. Toulouse): *Hyperbolic geometry to solve instances of Dehn's third problem, the isomorphism problem.*

The problem of algorithmically deciding whether two finite presentations define isomorphic groups is unsolvable (Adian, Rabin). However, in 1995 Sela proposed a solution to the isomorphy problem for a class of torsion-free rigid hyperbolic groups. The geometry of negative curvature allows to provide strong structural features for groups, and also gives possibilities to effectively compute in these groups (e.g. the problem of solving equations in these groups is algorithmically solvable, in a certain sense). In joint works with Daniel Groves, and Vincent Guirardel, we completed and extended Sela's program to a wide class of relatively hyperbolic groups, and to all hyperbolic groups. This work required revisiting an important algorithm of Makanin for solving equations in free groups, in a geometric and dynamical perspective.

Bettina Eick (TU Braunschweig): *The classification of p-groups by coclass.*

This is a survey on the recent advances on the classification of p-groups. It briefly discusses the classification of p-groups by their order. Then it introduces the idea of coclass and describes coclass graphs. Finally, it surveys some of the recent results on the classification by coclass.