Homework 1 **Due 3.03.2009**

Please save your Mathematica code in a .nb file and e-mail it to me.

1. Prove that if $f \in \mathbb{C}[x_1, \ldots, x_n]$ vanishes $(= 0)$ at every point of $\mathbb{Z}^n$, then f is the zero polynomial.

2.M Try to sketch the following affine varieties by hand and then use Mathematica to visualize them.

   1. $\mathbf{V}(xz^2 - xy)$;
   2. $\mathbf{V}(x^4 - zx, x^3 - yx)$;
   3. $\mathbf{V}((x-2)(x^2-y), y(x^2-y), (z+1)(x^2-y))$.

3. Let $R = \{(x, y) \in \mathbb{R} \mid y > 0\}$ be the upper half plane. Prove that R is not an affine variety.

4.M Write a program that computes the Greatest Common Divisor of two polynomials $p(x)$ and $q(x)$. You may use the Mathematica function PolynomialRemainder.

5. An ideal I is called a *radical* ideal when $f \in I$ if and only if $f^m \in I$ for some positive integer m. (a) Prove that $I(V)$ is a radical ideal for any variety V. (b) Prove that $\langle x^2, y^2 \rangle \neq I(V)$ for any variety $V \subset k^2$.