Homework 2 **Due 12.03.2009**

Please save your Mathematica code in a .nb file and e-mail it to me.

1.M If we use the grlex order with $x > y > z$, is $\{x^4y^2 - z^5, x^3y^3 - 1, x^2y^4 - 2z\}$ a Groebner basis? Why or why not? First check by hand and then check your answer using Mathematica.

2 Show that the Hilbert Basis Theorem is equivalent to the Ascending Chain Condition. That is, in every commutative ring, the two statements below are equivalent:

(a) every ideal $I \subset R$ is finitely generated,

(b) every ascending chain of ideals of R stabilizes.

3. Show that if G is a basis for an ideal I with the property that $\overline{f}^G = 0$ for all $f \in I$, then G is a Groebner basis for I.

4.M Write a program that computes the Groebner Basis of the ideal generated by a set of polynomials $(f_1, \dots, f_s)$ in two variables. You should follow Buchberger's algorithm (which we have done in class).

5. Let G and G' be Groebner bases for an ideal I with respect to the same monomial order in $k[x_1, \dots, x_n]$. Show that $\overline{f}^G = \overline{f}^{G'}$ for all $f \in k[x_1, \dots, x_n]$.